PRACTICE TEST SOLUTIONS

1) $$m = \lim_{h \to 0} \frac{\frac{3(x+h)}{(x+h)-9} - \frac{3x}{x-9}}{h}$$

$$= \lim_{h \to 0} \frac{(3x+3h)(x-9) - 3x(x+h-9)}{(x+h-9)(x-9)} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x^2} - \cancel{27x} + \cancel{3xh} - 27h - \cancel{3x^2} - \cancel{3xh} + \cancel{27x}}{h(x+h-9)(x-9)}$$

$$= \lim_{h \to 0} \frac{-27h}{h(x+h-9)(x-9)}$$

$$= \lim_{h \to 0} \frac{-27}{(x+h-9)(x-9)}$$

$$= \frac{-27}{(x-9)(x-9)}$$

at $x = 8$

$$m = \frac{-27}{(8-9)(8-9)}$$

$$= \frac{-27}{1}$$

$$= -27$$

To find pt. $f(8) = \frac{3(8)}{8-9}$

$$= \frac{24}{-1}$$

$$= -24$$

$\therefore (8, -24)$

$$y - (-24) = -27(x-8)$$

$$y + 24 = -27x + 216$$

$$y = -27x + 192$$

OR $0 = 27x + y - 192$

2.) $\lim_{x \to 2^-} f(x) = 2$

$\lim_{x \to 2^+} f(x) = 3$

$\lim_{x \to 2} f(x) = \text{DNE}$

$\lim_{x \to 4} f(x) = -3$

3a) $\lim_{x \to 6} 4x^2 - 3x$

$= 4(6)^2 - 3(6)$
$= 4(36) - 18$
$= 144 - 18$
$=$

b.) $\lim_{x \to 1} \frac{(x-5)(x-1)}{(x-2)(x-1)}$
$= \lim_{x \to 1} \frac{x-5}{x-2}$
$= \frac{1-5}{1-2}$
$= \frac{-4}{-1}$
$= 4$

c.) $\lim_{x \to 0} \frac{\sqrt{4-x} - 2x}{x}$
$= \frac{\sqrt{4-0} - 2(0)}{0}$
$= 0/0$ ∴ rationalize
$\lim_{x \to 0} \frac{\sqrt{4-x} - 2x}{x} \cdot \frac{\sqrt{4-x} + 2x}{\sqrt{4-x} + 2x}$
$\lim_{x \to 0} \frac{4 - x - 4x^2}{x(\sqrt{4-x}) + 2x^2}$
$= \frac{4}{0}$ ∴ DNE

3d.) $\lim_{x \to 4} \frac{3}{x^2-16}$

$= \lim_{x \to 4} \frac{3}{(x-4)(x+4)}$

V.A. @ $x=4$

$\therefore$ DNE

e.) $\frac{\frac{1}{3}-\frac{1}{x}}{x-7} = \frac{\frac{x-3}{3x}}{x-7}$

$= \frac{x-3}{3x} \cdot \frac{1}{x-7}$

$= \frac{x-3}{3x(x-7)}$

$\lim_{x \to 7} \frac{x-3}{3x(x-7)}$

DNE V.A. @ $x=7$

3f.) $\lim_{x \to 8} \frac{2-\sqrt[3]{x}}{8-x} = \frac{2-\sqrt[3]{8}}{8-8} = \frac{0}{0}$ $\therefore$ change variable

let $u=\sqrt[3]{x}$ $\therefore$ $u^3=x$

$u=\sqrt[3]{8}$

$=2$

$\lim_{u \to 2} \frac{2-u}{8-u^3} = \lim_{u \to 2} \frac{2-u}{(2-u)(4+2u+u^2)}$

$= \lim_{u \to 2} \frac{1}{4+2u+u^2}$

$= \frac{1}{12}$

4) $h(t) = -9.8t^2 + 6t + 30$

time when it hits the ground $(h(t)=0)$

$0 = -9.8t^2 + 6t + 30$

$= 9.8t^2 - 6t - 30$

$t = \dfrac{6 \pm \sqrt{(-6)^2 - 4(9.8)(-30)}}{2(9.8)}$

$= \dfrac{6 \pm 34.8}{19.6}$

$t_1 = 2.1s$ OR $t_2 = \cancel{-1.5s}$

$V(t) = \lim_{h\to 0} \dfrac{-9.8(t+h)^2 + 6(t+h) + 30 - (-9.8t^2 + 6t + 30)}{h}$

$= \lim_{h\to 0} \dfrac{-9.8(t^2 + 2th + h^2) + 6t + 6h + 30 + 9.8t^2 - 6t - 30}{h}$

$= \lim_{h\to 0} \dfrac{\cancel{-9.8t^2} - 19.6th - 9.8h^2 + \cancel{6t} + 6h \cancel{+30} \cancel{+9.8t^2} \cancel{-6t} \cancel{-30}}{h}$

$= \lim_{h\to 0} \dfrac{\cancel{h}(-19.6t - 9.8h + 6)}{\cancel{h}}$

$= -19.6t + 6$

velocity at 2.1s

$V(2.1) = -19.6(2.1) + 6$

$= -35.16 m/s$

5) $m = \lim_{h \to 0} \frac{(x+h)^2 + (x+h) - 5 - (x^2 + x - 5)}{h}$

$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{+x} + h \cancel{-5} \cancel{-x^2} \cancel{-x} \cancel{+5}}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(2x + h + 1)}{\cancel{h}}$

$= \lim_{h \to 0} 2x + h + 1$

$= 2x + 1$

at $x = -3$

$m = 2(-3) + 1$
$= -6 + 1$
$= -5$

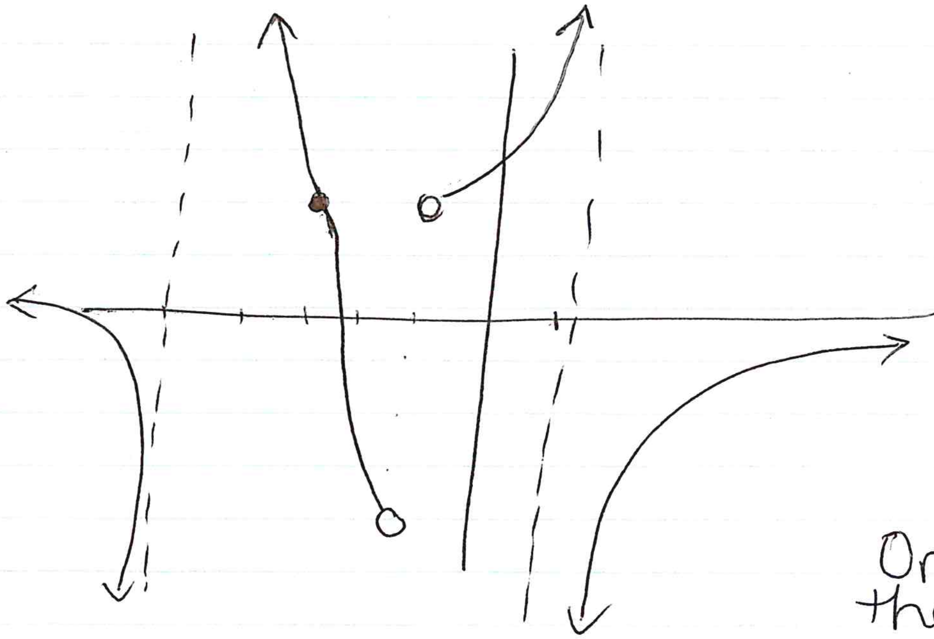
$y = (-3)^2 + (-3) - 5$
$= 9 - 3 - 5$
$= 1$

$\therefore (-3, 1)$

$y - 1 = -5(x - (-3))$

$y = -5(x+3) + 1$
$y = -5x - 15 + 1$
$y = -5x - 14$

6.)



One possibility... there are many others.

7a.) $1 - 6(2)$

$= 1 - 12$

$= -11$

b.) $\frac{5(1)}{2(2)}$

$= \frac{5}{4}$

c.) $-3(1)$

$= -3$

8.) $y = x^3 - 3x$

$$m = \lim_{h \to 0} \frac{(x+h)^3 - 3(x+h) - (x^3 - 3x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{3x} - 3h - \cancel{x^3} + \cancel{3x}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2 - 3)}{\cancel{h}}$$

$$= 3x^2 + 3x(0) + 0^2 - 3$$

$$= 3x^2 - 3$$

HORIZONTAL means $m = 0$ ∴ $3x^2 - 3 = 0$ $\quad 3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

9) $f(x) = ax^2 + bx + c$

$f(0) = 1 \Rightarrow a(0)^2 + b(0) + c = 1$

$c = 1$

$f(1) = -7 \Rightarrow a(1)^2 + b(1) + 1 = -7$

$a + b = -8$ (1)

$f(-2) = 3 \Rightarrow a(-2)^2 + b(-2) + 1 = 3$

$4a - 2b = 2$

$2a - b = 1$ (2)

(1) $b = -8 - a$

(1) $\rightarrow$ (2): $2a - (-8 - a) = 1$

$2a + 8 + a = 1$

$3a = 1 - 8$

$3a = -7$

$a = -7/3$

$b = -8 - (-7/3)$

$= -8 + 7/3$

$= \frac{-24}{3} + \frac{7}{3}$

$= \frac{-17}{3}$

$\therefore f(x) = -\frac{7}{3}x^2 - \frac{17}{3}x + 1$

10) $m = \lim_{h\to 0} \frac{(x+h)^2 + (x+h) - 5 - (x^2+x-5)}{h}$

$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{x} + h - \cancel{5} - \cancel{x^2} - \cancel{x} + \cancel{5}}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(2x + h + 1)}{\cancel{h}}$

$= \lim_{h\to 0} 2x + h + 1$

$= 2x + 1$

at $x = -3$ $\quad m = 2(-3) + 1$

$= -6 + 1$

$= -5$

$m_\perp = \frac{1}{5}$

pt $(2, 4)$

$y - 4 = \frac{1}{5}(x - 2)$

$5y - 20 = x - 2$

$5y = x + 18$

$\boxed{y = \frac{1}{5}x + \frac{18}{5}}$ OR $\boxed{0 = x - 5y + 18}$